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Radiative Majorana Neutrino Masses

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We present new radiative mechanisms for generating Majorana neutrino masses, within an extension of the standard model that successfully generates radiative charged lepton masses, order by order, from heavy sequential leptons. Only the new sequential neutral lepton has a right-handed partner, and its Majorana mass provides the seed for Majorana neutrino mass generation. Saturating the cosmological bound of 50 eV with m_{ν_τ}, we find that m_{ν_μ} and m_{ν_e} could be at most 10^{-2}, and 10^{-3} eV, respectively. The electron neutrino mass may vanish in the limit of degenerate charged Higgs bosons. Unfortunately, $\nu_e - \nu_\tau$ mixing is also radiatively induced, and is too small for sake of solving the solar neutrino problem via the Mikheyev–Smirnov–Wolfenstein effect.

There is no fundamental principle requiring the neutrinos to be massless. However, if neutrinos are not exactly massless, their near masslessness is a real mystery. The so-called seesaw mechanism [1] was invented, usually in the context of grand unified theories (GUT), to give a natural framework for almost massless neutrinos. It also provides a natural framework for explaining the solar neutrino puzzle via the Mikheyev-Smirnov-Wolfenstein effect [2]. Recent neutrino counting experiments [3] show that there exists three and only three species of light neutrinos. On the other hand, direct search for new sequential leptons at Z^0 resonance have been conducted and yields the limits [3]

$$m_E,\ m_N \gtrsim M_Z/2, \tag{1}$$

where we denote new charge and neutral leptons as E and N, respectively, and sequential means that left-handed leptons come in weak doublets. Although there is as yet no evidence for more sequential leptons, it has been pointed out [4] that if they are found with weak scale masses, the traditional seesaw mechanism and $SO(10)$ based GUT theories could be in jeopardy, and one may also face a serious challenge with the solar neutrino problem. It is therefore of interest to investigate whether very small neutrino masses can still be naturally generated under such circumstances. In this regard, Babu and Ma [5] have built a radiative seesaw model in which the three known neutrinos acquire radiative Majorana masses through the exchange of two W bosons, and could provide a natural explanation of the solar neutrino problem. Recently, we have sucessfully constructed a Z_8 model [6] where the charged leptons acquire mass radiatively order by order, with all Yukawa couplings of order unity. The three known neutrinos have no right-handed counterparts. They are held strictly massless if one requires the Majorana mass m_R for the single right-handed neutral lepton N_R to vanish. In this letter, we remove this *ad hoc* condition and investigate the question of light neutrino Majorana mass.

Let us briefly review the model. With minimal "$3+1$" generations [4], there is only one right-handed neutral lepton N_R. Consider a discrete Z_8 symmetry ($\omega^8 = 1$). We assign both $\bar{\ell}_{iL} = (\bar{\nu}_{iL},\, \bar{e}_{iL})$ and e_{iR} to transform as ω^3, ω^2, ω^1, ω^4 for $i = 1-4$, respectively, while N_R

transforms as ω^4. The scalar sector consists of three doublets, $\Phi_0,\ \Phi_3$ and Φ_5, transforming as 1, ω^3 and ω^5, respectively. Thus, except for $E \simeq e_4$ and N, only nearest-neighbor Yukawa couplings are allowed,

$$
\begin{aligned}
-\mathcal{L}_{\rm Y} = {} & f_{44}\bar{\ell}_{4L}e_{4R}\Phi_0 + \tilde{f}_{44}\bar{\ell}_{4L}N_R\tilde{\Phi}_0 \\
& + f_{43}\bar{\ell}_{4L}e_{3R}\Phi_3 + f_{34}\bar{\ell}_{3L}e_{4R}\Phi_3 + \tilde{f}_{34}\bar{\ell}_{3L}N_R\tilde{\Phi}_5 \\
& + f_{32}\bar{\ell}_{3L}e_{2R}\Phi_5 + f_{23}\bar{\ell}_{2L}e_{3R}\Phi_5 \\
& + f_{21}\bar{\ell}_{2L}e_{1R}\Phi_3 + f_{12}\bar{\ell}_{1L}e_{2R}\Phi_3 \qquad + H.c.
\end{aligned} \tag{2}
$$

where $\tilde{\Phi} \equiv i\sigma_2\Phi^* = (\phi^{0*}, -\phi^-)$ as usual. We assume CP invariance for sake of simplicity.

If only $\langle\phi_0^0\rangle = v/\sqrt{2}$, E and N become massive and are naturally at v scale if $f_{44},\ \tilde{f}_{44} \sim 1$. The lower generation leptons remain massless at this stage, protected by the Z_8 symmetry. To allow for radiative mass generation, the Z_8 symmetry is *softly* broken down to Z_2 in the Higgs potential by Φ_3-Φ_5 mixing. Explicitly,

$$
\begin{aligned}
V = {} & \sum_i \mu_i^2\Phi_i^\dagger\Phi_i + \sum_{i,j}\lambda_{ij}(\Phi_i^\dagger\Phi_i)(\Phi_j^\dagger\Phi_j) + \sum_{i\neq j}\eta_{ij}(\Phi_i^\dagger\Phi_j)(\Phi_j^\dagger\Phi_i) \\
& + [\tilde{\mu}^2\Phi_3^\dagger\Phi_5 + \zeta(\Phi_0^\dagger\Phi_3)(\Phi_0^\dagger\Phi_5) + H.c.],
\end{aligned} \tag{3}
$$

where λ_{ij} and η_{ij} are symmetric. Note that the ζ term is Z_8 invariant, while the gauge invariant "mass" $\tilde{\mu}^2$ transforms as ω^2. Since only $\mu_0^2 < 0$, while μ_3^2 and $\mu_5^2 > 0$, $\phi_0^0 \to (v + H_0 + i\chi_0)/\sqrt{2}$, and $\phi_i^0 \to (h_i + i\chi_i)/\sqrt{2}$ for $i = 3,\ 5$. The quadratic part of V is

$$
\begin{aligned}
V^{(2)} = {} & \lambda_{00}v^2H_0^2 + \sum_{i\neq 0}\left(\frac{1}{2}(\mu_i^2 + \lambda_{0i}v^2 + \eta_{0i}v^2)(h_i^2 + \chi_i^2) + (\mu_i^2 + \lambda_{0i}v^2)|\phi_i^+|^2\right) \\
& + \tilde{\mu}^2(h_3h_5 + \chi_3\chi_5 + \phi_3^-\phi_5^+ + \phi_5^-\phi_3^+) + \frac{1}{2}\zeta v^2(h_3h_5 - \chi_3\chi_5).
\end{aligned} \tag{4}
$$

The standard Higgs boson H_0 couples only diagonally to heavy particles. The nonstandard scalars $(\phi_3^\pm, \phi_5^\pm)$, (h_3, h_5) and (χ_3, χ_5) mix via $\tilde{\mu}^2$ and ζ terms. Rotating by θ_+, θ_H and θ_A, we obtain the mass basis (H_1^+, H_2^+), (H_1, H_2) and (A_1, A_2), respectively. It is clear that $\sin\theta_+ \to 0$, $(\theta_A, m_{A_1}, m_{A_2}) \to (-\theta_H, m_{H_1}, m_{H_2})$ as $\tilde{\mu}^2 \to 0$, while in the limit $\zeta \to 0$, one has $(\theta_A, m_{A_1}, m_{A_2}) \to (+\theta_H, m_{H_1}, m_{H_2})$. These two limits restore the two extra $U(1)$ symmetries of the doublets Φ_3 and Φ_5.

Since N_R has Z_8 charge assignment of ω^4, a Majorana mass term m_R is in fact permitted. We have previously set this term to zero [6] for sake of simplicity. This was in part also for the reason of "naturalness", since cosmological considerations [7] suggest that $m_R/v \ll 1$, in strong contrast to other dimensionless parameters of the model. If we do permit nonzero m_R, however, we see that it breaks the chiral symmetry of the first three generation of massless neutrinos. It provides the seed for generating tiny Majorana neutrino masses through mixing and nondegeneracy of the four real scalar fields, as well as the two charged scalars.

Radiative mass generation for charged leptons has already been discussed in ref. [6]. Here, we concentrate on neutrino mass generation. The tau neutrino ν_τ acquires Majorana mass via the one-loop diagram shown in Fig. 1, which is very similar to the charged lepton mass generation mechanism. We find

$$(m_\nu)_{33} = \left(\frac{\tilde{f}_{34}^2}{32\pi^2}\right) \Big[\sin^2\theta_H\, G(m_{H_1}/m_R) + \cos^2\theta_H\, G(m_{H_2}/m_R) \\ - \sin^2\theta_A\, G(m_{A_1}/m_R) - \cos^2\theta_A\, G(m_{A_2}/m_R)\Big]\, m_R, \tag{5}$$

where $G(x) = (x^2 \ln x^2)/(x^2 - 1)$, while $(m_\nu)_{34} = (m_\nu)_{43} = 0$. Note that the loop-induced Majorana mass is proportional to m_R.

Light neutrinos are subject to the cosmological upper bound [7] of 50 eV on their masses, unless they have fast decay channels. In absence of the latter, we saturate this bound with m_{ν_τ}, assuming that the quantity in brackets is of order one, and that all Yukawa couplings are of order unity in this model [6]. We find that the tree level Majorana mass m_R should be less than a few hundred keV. This may appear unnatural at first sight. We will return for discussions later.

The muon neutrino ν_μ acquires mass via the exchange of two $\phi_5^\pm$ bosons (in gauge basis) with overlapping loops, as shown in Fig. 2. Because the three left-handed neutrinos do not have right-handed partners, one cannot have analogous two-loop nested diagrams as for charged lepton mass generation [6]. Fig. 2 is generated as follows. From nearest neighbor Yukawa couplings, which follow from the Z_8 charge assigments, the left-handed ν_μ first couples to a right-handed τ via the f_{23} term. An m_τ insertion is necessary to flip the

chirality of τ, before another charged ϕ_5 scalar emission flips τ_L into N_R via the $\tilde{f}_{34}$ term. After m_R insertion, the sequence is repeated in reverse order. In all, two m_τ insertions are necessary, but ultimately m_R provides the seed. It is also clear that only the $\phi_5^\pm$ boson enters the diagram, hence the process does not depend on scalar mixing in any crucial way. The contribution from Fig. 2 is

$$(m_\nu)_{22} = -(f_{23}\tilde{f}_{34})^2\, m_R\, m_\tau^2 \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m_\tau^2}\left[\frac{s_+^2}{p^2-m_{H_1^+}^2}+\frac{c_+^2}{p^2-m_{H_2^+}^2}\right]\times$$
$$\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m_\tau^2}\,\frac{1}{(p+q)^2}\left[\frac{s_+^2}{q^2-m_{H_1^+}^2}+\frac{c_+^2}{q^2-m_{H_2^+}^2}\right]. \tag{6}$$

In the above s_+^2 and c_+^2 are short-hand for $\sin^2\theta_+$ and $\cos^2\theta_+$, respectively, and we have also ignored m_R in the denominator since $m_R \ll m_\tau$. This approximation simplifies the calculation. The presence of two tau mass insertions makes the diagram finite. Note that the two-loop Dirac mass term $(m'_\nu)_{24}\bar{\nu}_{\mu L}N_R$ can be rotated away and does not contribute to m_{ν_μ} directly, but modifies the Dirac mass for N slightly. Since each power of m_τ is at one-loop order [6], Fig. 2 is effectively at four-loop order. In contrast, the contribution from the exchange of two W bosons [5] is effectively at six-loop order within the model, since mixing in leptonic charged current is also loop induced [6]. Hence, numerically it can be neglected in our model.

Eq. (6) can be reduced to

$$(m_\nu)_{22} = -\left(\frac{f_{23}\tilde{f}_{34}}{16\pi^2}\right)^2 m_R\, m_\tau^2 \sum_{i,j=H_1^+,H_2^+}\frac{g_{ij}^{(\mu)}}{(m_i^2-m_\tau^2)(m_j^2-m_\tau^2)}\times$$
$$\int_0^1 dx\{m_j^2[-Li_2(-u)]_{u=(a_{\tau j}-x)/x}^{u=(a_{ij}-x)/x} - m_\tau^2[-Li_2(-u)]_{u=(1-x)/x}^{u=(a_{i\tau}-x)/x}\}, \tag{7}$$

where

$$g_{ij}^{(\mu)} = \begin{pmatrix} s_+^4 & s_+^2c_+^2 \\ s_+^2c_+^2 & c_+^4 \end{pmatrix},$$
$$a_{xy} = m_x^2/m_y^2, \quad (x,y=i,j,\tau). \tag{8}$$

Here, $Li_2(u)$ is the dilogarithm (polylogarithmic function of order 2). The integral in eq. (7) is evaluated numerically. Note that the Majorana mass $(m_\nu)_{24}$ is of the same loop order as

$(m_\nu)_{22}$. Nevertheless, it is still reasonable to use $(m_\nu)_{22}$ to represent m_{ν_μ} since these higher order masses are much less than the tree level mass m_R. To estimate the range of m_{ν_μ}, we will take the maximal value $m_R = 0.5$ MeV (the electron mass!) in eq. (7). As mentioned in ref. [6], the nonstandard Higgs bosons cannot be too far above the electroweak scale while all Yukawa couplings should be of order unity. We therefore take $f_{23} = \tilde{f}_{34} = \sqrt{2}$ and fix $\sin\theta_+ = 0.2$. We plot in Fig. 3 m_{ν_μ} versus $m_{H_2^+} \in (125,\ 4000)$ GeV, for $m_{H_1^+} = 125$, 250, 500, 1000 GeV. We see that m_{ν_μ} can reach up to the order of 10^{-2} eV. It is likely to be within the range of $10^{-3} - 10^{-2}$ eV for $m_{H_2^+}$ below 2 TeV. In general, it is insensitive to $m_{H_1^+}$ if $\sin\theta_+$ is not too large, since Fig. 2 is due to $\phi_5^\pm$ alone in gauge basis.

The electron neutrino ν_e acquires mass via the exchange and *mixing* of ϕ_3^- and ϕ_5^-, as shown in Fig. 4. The left-handed electron neutrino flips to a right-handed muon by emitting a ϕ_3^+ boson via the f_{12} coupling. The right-handed muon then emits a ϕ_5^- boson and changes into left-handed tau neutrino. Since ν_τ has acquired Majorana mass via Fig. 1, m_{ν_τ} now effectively functions as seed. The sequence is then repeated in reverse order. It is clear that $\phi_3^\pm$–$\phi_5^\pm$ mixing is needed. Since m_{ν_τ} is effectively at one-loop order, this is effectively a three-loop diagram. The contribution from Fig. 4 is

$$(m_\nu)_{11} = -(f_{12}f_{32})^2\, s_+^2 c_+^2\, m_{\nu_\tau} \int \frac{d^4p}{(2\pi)^4}\frac{\not{p}}{p^2}\left[\frac{-1}{p^2 - m^2_{H_1^+}} + \frac{1}{p^2 - m^2_{H_2^+}}\right] \times$$
$$\int \frac{d^4q}{(2\pi)^4}\frac{\not{q}}{q^2}\,\frac{1}{(p+q)^2}\left[\frac{-1}{q^2 - m^2_{H_1^+}} + \frac{1}{q^2 - m^2_{H_2^+}}\right], \qquad (9)$$

where we have ignored m_{ν_τ} in the denominator since $m_{\nu_\tau} \ll m_{H_1^+}, m_{H_2^+}$. After some calculation the above equation becomes

$$(m_\nu)_{11} = \left(\frac{f_{12}f_{32}}{16\pi^2}\right)^2 s_+^2 c_+^2\, m_{\nu_\tau} \sum_{i,j=H_1^+,H_2^+} \frac{g_{ij}^{(e)}\, m_j^2}{m_i^2} \times$$
$$\int_0^1 x dx\, \left[-Li_2(-u)\right]_{u=-1}^{u=(a_{ij}-x)/x}, \qquad (10)$$

where

$$g_{ij}^{(e)} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \qquad (11)$$

The diagram is logarithmically divergent, but the GIM-like mechanism of $\phi_3^\pm$–$\phi_5^\pm$ mixing renders the contribution finite. To evaluate the plausible range for m_{ν_e}, we take $f_{12} = f_{32} = \sqrt{2}$, $m_{\nu_\tau} \sim 50$ eV, and plot, in Fig. 5, m_{ν_e} versus $m^+_{H_2}$ with $m^+_{H_1} = 125$, 250, 500 1000 GeV, fixing $\sin\theta_+ = 0.2$. We see that m_{ν_e} can reach up to order 10^{-3} eV if H_1^+ and H_2^+ are far from being degenerate. However, as expected, m_{ν_e} vanishes for $m_{H_1^+} = m_{H_2^+}$ and we see that this GIM-like cancellation effect is rather strong, and m_{ν_e} could be far below 10^{-3} eV.

Some discussion is in order. It may be asked why the effectively four-loop ν_μ mass generation diagram is seemingly larger than the effectively three-loop ν_e mass generation diagram. The reason is because Fig. 2 depends on $\phi_5^\pm$ alone, and is not sensitive to charged scalar mixing. Indeed, we have taken the mixing angle $\sin\theta_+$ as "small" and the result depends largely on $m_{H_2^+}$, the physical charged scalar that is mostly the ϕ_5^+ in gauge basis. In contrast, the mechanism in Fig. 4 for ν_e mass generation depends crucially on $\phi_3^\pm$–$\phi_5^\pm$ mixing. It provides a GIM-like mechanism that not only leads to divergence cancellation, it further suppresses the three loop contribution in an interesting parameter domain. From the prespective of the model, H_1^+ and H_2^+ mass should not be too widely apart [6], hence for the plausible range of $m_{H_{1,2}^+} \in (0.5,\ 2)$ TeV, m_{ν_e} is rather suppressed. But outside of this domain, if H_1^+–H_2^+ splitting is rather sizeable, one in general finds Fig. 4 dominating over Fig. 2, as expected.

Second, although all light neutrino Majorana masses vanish with m_R, note that ν_μ mass is directly related to m_R and suffers from no cancellation mechanism. In contrast, m_{ν_τ} and m_{ν_e} have separate cancellation mechanisms in the form of scalar mixing, and in fact m_{ν_e} depends on m_R through an ν_τ mass insertion. The reason behind the difference between m_{ν_μ} *vs.* m_{ν_e}, m_{ν_τ} is the remnant Z_2 symmetry [6] after soft-breaking of Z_8: ν_μ and N_L is even, while ν_e and ν_τ are odd, as are the nonstandard scalar bosons. Thus, ν_e and ν_τ belong to the same class, while ν_μ is more closely related to N_L. This dichotomy of leptons is very useful in suppressing dangerous tranistions like $\mu \to e\gamma$. Third, it is of interest to look into questions related to neutrino oscillations. Inspection of Fig. 4 suggests that one has a ν_e–ν_τ Majorana mixing term also of three-loop order. The only difference is that for one of

the scalar loops, one has $\phi_5^{\pm}$ throughout, hence one is less sensitive to the GIM like ϕ_3^+–ϕ_5^+ mixing effect, although it is still needed for sake of divergence cancellation. Note however that this off-diagonal term is still at three-loop order and proportional to the one-loop m_{ν_τ} mass. Hence, we estimate the ν_e–ν_τ mixing angle to be of order $10^{-5}-10^{-4}$, which is almost independ of m_{ν_τ} value, but depends on the fact that there are two extra loops compared to Fig. 1. The mass eigenvalue is barely changed from that of eq. (10). The mass difference between ν_e and ν_τ is basically just m_{ν_τ}. We have used it to saturate the cosmological bound, but it can be lowered if one is willing to entertain a lower m_R value. However, since the tiny mixing angle is more or less independent of m_{ν_τ}, we find that our model cannot provide a basis for a MSW-type solution to the solar neutrino problem. Finally, we have to face the issue of naturalness of the m_R scale. One of the most interesting aspects of our radiative model for charged lepton mass generation [6] is that all dimensional parameters are of order weak scale, while all dimensionless parameters are of order one. To allow for Majorana neutrino mass generation, however, we need m_R to be less than one MeV, which seems to run against the spirit of the model. We note, however, that m_R in the model is a very different parameter from all the others. It is not related in anyway to the Higgs bosons introduced. It is basically a free parameter in the same way that m_e is a parameter for QED. It is clear that setting it to zero one restores a larger chiral symmetry. We therefore would like to advocate a lenient attitude towards the naturalness question for m_R.

In summary, if new sequential leptons are found at the weak scale, the traditional seesaw mechanism for neutrino mass generation, as well as $SO(10)$ based GUT theories, may be in jeopardy. One may therefore face a serious challenge with the solar neutrino problem. We provide a new radiatve mechanism for generating tiny Majorana neutrino masses. Because of remnant Z_2 symmetry, ν_τ and ν_e receive Majorana mass in a different way than ν_μ, although in each case the tree level Majorana mass for heavy neutral lepton, m_R, provides the seed. If one saturates the cosmological bound with m_{ν_τ}, then m_{ν_μ} and m_{ν_e} are typically of order 10^{-2} and 10^{-3} eV, respectively, although m_{ν_e} can be considerably smaller. Unfortunately, $\nu_e - \nu_\tau$ mixing is loop induced and is too small to sustain an MSW solution to the solar

neutrino problem.

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FIGURES

FIG. 1. Radiative mechanism for m_{ν_τ}.

FIG. 2. Radiative mechanisms for m_{ν_μ}.

FIG. 3. m_{ν_μ} *vs.* $m^+_{H_2}$ for (top to bottom) $m^+_{H_1} = 125$, 250, 500 1000 GeV at fixed $\sin\theta_+ = 0.2$.

FIG. 4. Radiative mechanisms for m_{ν_e}.

FIG. 5. m_{ν_e} *vs.* $m^+_{H_2}$ for (top to bottom) $m^+_{H_1} = 125$, 250, 500 1000 GeV at fixed $\sin\theta_+ = 0.2$.